Optimization of Bus Routes in Small Cities Based on Dijkstra's Algorithm (2024)

Abstract: In order to solve the problem of irrational planning of bus routes within a small city, which cannot meet the daily public travel demand of residents, a bus path optimization model considering passengers' travel time is proposed. Under the conditions of known passenger flow between bus stops, vehicle passenger capacity, frequency of departure, etc., the existing bus routes are optimized according to the passenger demand, and the minimization of the operating cost of vehicles (vehicle driving time, fuel consumption, etc.) and the time cost of passengers (time waiting for vehicles before boarding the bus, ride time, arrival time, etc.) is taken as the objective, the bus path optimization model considering passenger satisfaction is constructed, and the Dijkstra algorithm is used to solve the problem. Dijkstra algorithm for solving. Finally, the feasibility of the model and algorithm is verified through the example analysis of Fangchenggang Fangcheng District bus route. The results show that in the route optimization of 103 bus stops in Fangchenggang City, the number of routes is reduced by two, but the density of the line network is increased by 10.3%, the non-linear coefficient is reduced by 6.3%, and the passenger satisfaction is improved. Greater satisfaction of passenger demand, effective reduction of errors in planning paths, shortening of travel distance and passenger travel time, and increased passenger satisfaction. Compared to the original bus routes, the model results considering passenger satisfaction are better.

Keywords: Bus, Dijkstra algorithm, Route Optimization, Urban Transportation.

1. Introduction

Urban buses play an important role in the public transportation system, providing daily travel services for urban residents. According to the national urban passenger transportation volume published by the Ministry of Transportation and Communications in January-February 2024, the urban passenger transportation volume of public buses and trams was 60,181,200,000 passengers, accounting for 36.4% of the total urban passenger transportation volume, and this ratio is even higher in small cities without subways. With the development of the city, the public transportation system is gradually formed and improved, at the same time, due to the stage of urban development and geographical, making public transportation in the process of development, excessive attention to the city's circulatable situation and ignored the degree of excellence of its performance. At present, most of the urban public transport system does not have a systematic management, especially in some small cities, the bus network of the clutter and the low utilization rate of the line code makes the lack of holistic planning of the public transport system, but not to improve the efficiency of the people's travel, to some extent, but on the city's transportation and residents travel to cause certain adverse effects. When choosing a mode of transportation, passengers usually focus on factors such as time consumption, time reliability and comfort. Time consumed includes waiting time for the ride and the duration of the ride. Time reliability is critical for activities such as commuting to work and school; and comfort relates to how crowded the bus is. With rapid urbanization, cities are expanding in size and require corresponding urban infrastructure. However, in many small cities, especially in the old urban areas, the bus operation system still has not been sufficiently improved, and there are many irrationalities in the operation routes, which include insufficient routes and limited coverage, resulting in a number of areas not being able to enjoy the convenience brought by public transportation, especially in remote areas or newly built areas; the poor quality of service, inaccurate arrival times, and unstandardized driving behavior, which reduces the overall service level of the public transportation system and increased passenger’s travel time. This results in ineffective connections between old and new urban areas, and passengers need to make frequent transfers, increasing travel time and inconvenience. As a result, more and more people choose to give up public transportation.

According to the above existing problems, the city's bus operation and development is facing some challenges, if not make changes, will face problems such as bus shutdown. Through the optimization of bus routes, the level of bus service can be improved to meet the travel needs of residents and enhance the operation of the bus system. This will help reduce the use of private cars, ease traffic congestion and promote sustainable urban development. The problem of bus path optimization is also widely concerned and researched by scholars and engineers, and a reasonable bus path optimization scheme can not only save the cost of the bus company, but also improve the travel convenience and satisfaction of the residents.

Urban public transportation is a public welfare undertaking to provide basic travel services for the public, and it is a major livelihood project related to the people's "food, clothing, housing and transportation"; on the other hand, as a transportation enterprise to provide urban public transportation services, it should improve its own operational efficiency [1]. Residents in the city are mainly traveling in short and medium distances, and cabs are highly flexible but have a small capacity; subways have a large capacity but are not flexible enough. Urban bus system has a wide coverage, large capacity, flexibility and other characteristics [2]. However, at present, many small cities have problems such as too long bus lines, more bus stops along the line, and uneven number of passengers at some bus stops along the line [3].

At present, scholars at home and abroad for the bus path optimization problem research, mainly through the establishment of a single-objective or multi-objective function, additional constraints optimization model, and then use the corresponding algorithms for solving. And at present, most of the influencing factors for the model are stops, passenger flow, line length, speed, number of vehicles, etc., and most of the objective function is the travel cost of passengers, the government's public operating costs, etc. Li Xin et al. (2023) in the conventional bus route optimization on the basis of the influence of factors, more consideration of passenger travel time factors, the passenger arrival at the station i and the departure time of the passenger at the station i as an influence of factors, and at the same time, set the latest arrival time, give full consideration to the passengers to plan the bus route [4]. Chao Wang et al. (2020) mainly consider the impact of the flat peak on the flow of people, the time of day is divided into a number of flat peaks, the time of day is divided into a number of flat peaks, the time of day is divided into a number of flat peaks. The time of the day is divided into several flat peaks and peaks, with different optimization strategies for different time periods, which can further save the cost of the bus company [5].Yue Zheng et al. (2021) also add the most constraints of service quality to the model, which includes punctuality and comfort, etc., which are added to the cost of the bus company [6].Rui Oliveira et al. ((2019), on the other hand, from the perspective of buses, through the fuel consumption of buses at different speeds, different road conditions, etc., which are added to the constraints of the model to optimize the paths in order to reduce the fuel consumption to reduce costs [7]. He Xueding et al. (2023) take the passengers as the center, add the earliest departure time and the latest arrival time of passengers into the model, in order to satisfy the travel needs of all passengers through optimization [8].Ziling Zeng et al. (2023) do the path optimization problem of electric buses, which mainly takes the location of the charging piles and the charging speed of the electric buses into account in the model, and takes them as a passenger boarding point that needs a waiting time. waiting time for passenger boarding point to optimize the electric bus path [9]. Guan Deyong et al. (2022) argued that buses can determine some mandatory boarding and alighting points according to the demand, and optional boarding and alighting points, that is, some bus stops can be made without stopping, which can optimize the bus travel time [10].Ahmed Tarajo Bub et al. (2019) mainly took into account the minimum permissible frequency of the buses, the maximum traffic flow on the key routes, the maximum and minimum route lengths to constrain the planned bus routes [11], and the maximum and minimum route lengths [12]. aspects to constrain the planned bus routes to ensure that the bus routes are more realistic [11]. Chigechi (2022) et al. added penalty costs to the model, including punctuality, speed and comfort penalty costs, and these are then used as bus operating costs, which makes the bus route optimization need to be further optimized in order to ensure the profitability of the bus company [12].S. M. Mahdi Amiripour et al. (2014) also added the waiting time and ride time of the passengers as costs to the model and also considered passenger dissatisfaction, which was added as a penalty factor to make the bus company more concerned about the interests of passengers [13]. Wu Hongbo et al. (2022) reduced passenger travel time mainly by reducing the number of transfers by converting the number of transfer stops into the number of stops needed to be ridden, which further reduces the number of stops needed to be ridden through algorithmic optimization and reduces the passenger travel time [14].Changxi Ma et al. (2021)

added the carbon emission of the bus into the constraints of the path optimization model and compared it with the passenger load of the bus to reduce the travel time. and correlates it with the number of passengers carried by the bus to find an optimized path with minimum carbon emission that meets the capacity requirements [15]. Guan Yunlin et al. (2021) optimized the path of event buses and mainly considered the relationship between the number of people in demand, bus vehicles and full load rate to satisfy the capacity to provide as much comfort as possible as the event buses need to be on time and comfortable [16].Edgar Ruano-Daza et al. (2018) also mainly considered the number of people in the bus and other factors, and added the number of people in the bus in the previous stop as an influencing factor added to the model, which has an impact on the number of available boardings and ride comfort at the later stations [17]. Hu Yucong et al. (2018) based on the theory of conventional bus planning and combined with the operational characteristics of customized buses, constructed an optimization design model for customized bus routes with multiple stops, multiple vehicle types, and mixed loading of various types of passengers [18]. Cordeau (2006) aimed at the route optimization problem of dial-a-ride travel, established a mixed integer programming model with vehicle capacity, time window, priority, etc. as constraint conditions, and adopted the branch-and-cut algorithm to solve the model [19]. Quadrifoglio et al. (2008) in the early stage, through the analysis of the key parameters of the flexible bus system, proposed a mixed integer programming model to solve the route optimization problem of the mobility allowance shuttle transit (MAST) system, and established a series of logical sets by using reasonable assumptions of passenger behavior to accelerate the search for the optimal performance [20].

In the path optimization problem, many scholars have also paid attention to the road network design problem, and its research has a history of several decades of development. In 1973, Morlok first proposed the quantitative road network design problem and carried out research on it, forming a new research direction in the field of transportation planning ---- the network design problem(NDP). LeBlanc (1975) first began to systematically study the urban road traffic network design problem and represented the network design problem with a mixed integer optimization model [21]. Abdulaal et al. (1979) improved on the previous first-author work, put forward a new urban road traffic network design model, and used continuous variables as decision variables. In this model, the route choice behavior of travelers in the network is represented by equilibrium flow constraints [22]. This model is the originator of the continuous equilibrium network design model for urban road traffic. Lin J J et al. (2003) have studied the combined problem of land use and traffic network design. They believe that improving land use efficiency and classifying land use are the keys to optimizing the traffic network design problem [23]. Liu Canqi (2003) taked into account the reaction of transportation hubs to the transportation network, proposed a bi-level decision-making model for the simultaneous optimization of the location selection of transportation hubs and network design [24]. Li Xiugang et al. (2004) have proposed a signal timing model for road network intersections. By using comprehensive optimization methods, it can minimize the delay time, exhaust emissions, and energy consumption of a single intersection to the greatest extent. However, this model only takes into account the exhaust emission control of a single intersection rather than the entire traffic network [25]. Liu Weiming (2004) introduced a bilevel programming method to describe the positions, goals, and decision-making functions of highway network managers, toll road operators, and road users in network design issues. It comprehensively optimizes the location and capacity of toll sections on highways, the scale of investment, and the toll rates of toll roads, and has conducted a relatively comprehensive study on the equilibrium network design problem of highways [26]. Sumalee et al. (2004) have considered the network design problem under the condition of demand variation, which I call the reliable network design problem. Although a network design model based on maximizing the reliability of the total travel time has been proposed, its algorithm and the model are merely a conceptual framework and cannot be applied in practice yet [27]. Chootinan et al. (2005) introduced capacity reliability into the urban road network design problem. However, only the daily travel situations of travelers were taken into account, while the network characteristics when travel demand changes or the system capacity deteriorates were not considered [28].

The optimization of urban bus routes is a crucial issue for enhancing the quality of public transportation services and promoting the sustainable development of cities. Existing research has made certain progress in bus route optimization and road network design.

In the research on bus route optimization, scholars have established optimization models by constructing single - objective or multi - objective functions and adding constraint conditions, and solved them using various algorithms. These studies have considered multiple influencing factors such as stops, passenger flow, route length, speed, number of vehicles, etc. The objective functions cover aspects such as passenger travel costs, government operating costs, and the profitability of bus companies. For example, some studies focus on passenger travel time, the impact of passenger flow during different periods, service quality constraints, vehicle fuel consumption, special time requirements of passengers, bus charging factors, optimization of stop settings, route planning constraints, cost penalty mechanisms, reduction of transfer times, carbon emission control, and route planning for different types of buses.

The research on road network design has also experienced long - term development. Starting from the early proposal of quantitative research questions, the models have been continuously improved, considering factors such as land use, the impact of transportation hubs, signal timing, toll system optimization, and network reliability. However, current research still has certain limitations, especially in the combination of theory and practice. There is a relative lack of achievements that can effectively guide the actual bus route planning.

Based on this, this paper proposes a bus route optimization model considering passenger travel time, solves it using the Dijkstra algorithm, and verifies it through an example of the bus routes in Fangcheng District, Fangchenggang City. The aim is to provide a new and effective solution to the problem of irrational bus route planning in small cities, and to improve the operational efficiency and service quality of the bus system.

2. Bus Path Optimization Model

2.1. Problem Description

According to the spirit of the Circular on Adjusting the Criteria for Classifying the Size of Cities issued by the State Council of the People's Republic of China, small cities are those with a permanent urban population of less than 500,000 people. The difference in city size also leads to a big difference between the travel demand of small cities and that of big cities, mainly in four aspects: the number of trips, the distribution of travel purposes, the mode of travel, and the distance of travel. And due to the different regional development process, small cities also show obvious differences within the area.

In recent years, small city transit systems have also faced difficulties. Many parts of the country bus system losses are serious, facing the shutdown, small city bus lines would have been relatively small, bus shutdown is often more than one line together, and even the whole city bus shutdown occurs, seriously affecting the daily travel of urban residents. The main reason is that small cities can not afford to subsidize, small cities are many county-level cities, compared to large cities, small counties have limited local financial resources, it is difficult to provide adequate subsidies to the bus company, becoming an important factor in the public transport enterprises can not make ends meet. According to the National Transportation Industry Development Statistics Bulletin, affected by the sharing economy such as small cars and net car, coupled with the outbreak of the new crown epidemic in recent years, the residents to reduce the awareness of gathering, the city passenger traffic has declined year by year, with the highest rate of decline as high as 31.8%. Small city bus ridership is small, bus companies can not make ends meet another reason for the decline in ridership, want to save the small city's public transport plight, we must attract residents to re-option of bus travel. When choosing a means of transportation, riders are concerned with time spent, time reliability, and comfort. Time spent includes waiting time for a ride and time spent riding the bus. Passengers are also concerned about time reliability when choosing a mode of transportation, as they need to be assured of time to get to work and school. Comfort is mainly concerned with the degree of crowding on the bus. Over the past few decades, China has experienced rapid urbanization and expansion of its cities, which requires supporting urban infrastructure. But many small cities in the public transportation system has not changed much, especially in the old city, still using the previous bus operating system, operating lines, which there are many unreasonable places. The most important performance here is the line, with the construction of new urban areas, the center of gravity of the city has changed, the bus route so that the new and old cities can not be effectively connected to the bus transfer line makes the bus travel time-consuming and laborious, so more and more people give up the bus travel, choose private car travel, exacerbating the

road congestion. So if you want to attract more people to take the bus, you need to optimize the existing bus routes, make the bus routes more humane, meet the needs of public travel, and provide a comfortable and uncrowded riding environment.
The operation of buses in small cities also has the following two characteristics:
Simple route layout: Given the relatively small urban areas, bus routes are usually not as complicated as those in large cities. The number of stops is limited and they are relatively concentrated. Most routes mainly cut through main streets and connect key areas, like the city center, residential areas, schools, and hospitals. Overall, the layout is more compact and concise.
Shorter operating mileage: The small size of the city dictates that the mileage of a single bus journey is generally short, mostly falling within the range of a few kilometers to over ten kilometers. Ultra-long-distance bus routes rarely occur, which also leads to a relatively high vehicle turnover rate.
Bus route optimization usually belongs to a class of problems called Public Transit Route Planning Problem. This type of problem can be further subdivided into different subproblems, and the problem studied in this paper is one of the main subproblems: Bus Routing Optimization. Determining how buses travel on roads in an urban network to serve passenger demand and minimize costs usually takes into account factors such as vehicle capacity, stop capacity, and traffic conditions. Bus route planning is the problem of determining bus routes and travel paths to optimize some objective function (e.g., passenger travel time, vehicle operating costs, etc.) in a given network of stops.

2.2. Model Assumptions

To simplify the research problem while still reflecting the real situation, the following assumptions are proposed:

Buses maintain a constant speed during the driving process.

The road conditions are good, and special situations such as traffic congestion and traffic accidents are not taken into consideration.

All buses have the same vehicle type and passenger-carrying capacity.

Based on the scale of the research area in this paper, only direct bus routes are considered, and transfer routes are not taken into account.

2.3. Variable Assumptions

In this problem, we assume the following variables: the number of total bus stops n; the total number of bus routes m; a decision variable x_{ijk} for whether the point i and j stops are connected to each other via the kth route, with $x_{ijk} = 1$ when connected and $x_{ijk} = 0$ otherwise; the frequency of bus departures f_k; the number of passengers from i to j in route k, p_{ijk}; the passenger boarding and alighting time, h; and the fuel consumption of the bus per unit of trip, c; The length of the point m line l_m; the bus fare q; the government subsidy to the bus company c_g; the average speed of the bus v; the number of stops n_r on the rth line; the average stopping time of the bus at the stops t_n; the departure time of the eth bus t_e; the distance from the ith bus stop to the jth bus stop d_{ij}; and b_{ijk} is a decision variable, when the kth bus travels from the point i to the j stop, the b_{ijk}=1, otherwise=0; z_{ije} denotes the number of people that the eth bus receives from the point i stop to j stop; P is the set of passengers; B denotes the bus capacity; β denotes the upper limit of the full load rate; and N denotes the number of departures per day. The meaning of each symbol is shown in the table below:

Table 1. Symbol Meanings.

Symbol	Meaning
i、j	Indicates the i, j stop
n	Indicates the total number of bus stops
k	Indicates the point m line

m	denotes the total number of bus lines
x_{ijk}	Stations i and j are connected by route k, xijk=1 otherwise=0
s_{ij}	Stations i and j can be connected by transfer, sij=1 otherwise=0
f_k	denotes the frequency of bus departures
p_{ijk}	denotes the number of passengers from i to j in line k
h	denotes the passenger boarding and alighting time
c_s	denotes the fuel consumption of the bus per unit trip
l_m	denotes the length of the point m route
q	denotes the bus fare
c_g	denotes the operating cost per unit of the bus
v	denotes the average speed of the bus
n_r	denotes the number of stops on route r
t_n	denotes the average stopping time of the bus at the stops
t_e	denotes the departure time of the eth bus
d_{ij}	denotes the distance from the point i bus stop to the point j bus stop
b_{ije}	When the point e bus travels from the point i stop to the point j stop, bije = 1, otherwise = 0;
z_{ije}	denotes the number of people that the eth bus received from the point i stop to the point j stop
P	denotes the set of passengers
B	denotes the number of passengers carried by the bus
β	denotes the bus load factor
N	denotes the number of bus trips per day

The bus route planning problem involves several variables and parameters and requires the design of an optimization model to optimize the objective function. By reasonably selecting and constraining these variables and parameters and using appropriate optimization algorithms, we can find the optimal bus routes and travel paths to improve the efficiency and service quality of the bus system.

The optimization of urban bus routes involves the interests of two parties, passengers and bus operating companies. If we want to optimize the bus routes, we should consider from these two aspects, take into account the interests of both sides, and balance the interests of both sides. First of all, passengers take the bus is mainly concerned about the time consumption, want to reduce travel costs, passengers also need public transport has a certain time reliability, try to arrive at their destinations on time, the time reliability of the bus and the total time of the passenger bus ride on the passenger willingness to take the bus has a greater impact. Considering the economic principle, there is no need to rebuild bus stations and introduce a large number of new buses, so try to use the original stations.

The frequency of bus departures on the kth line is f_k, so the waiting time for a passenger to ride the kth line is $\frac{1}{2f_k}$.

The number of passengers arriving at station j $\{a_j(t)\}$ is a stochastic process whose mean process is $\{\mu_j(t)\}$, j=1,2,3，…， n; the number of passengers leaving station j $\{b_j(t)\}$ is a stochastic process whose mean process is $\{v_j(t)\}$, j=1,2,3， …， n.

The moment when the point k bus leaves stop j is

$$T_{k,j}=\begin{cases} t_k, & j=1,\ \ i=1,2,\ \ \dots,\ \ n, \\ t_k+\frac{d_{1j}}{v} & j=2,3,\dots n, i=1,2,\dots n; \end{cases} \quad (1)$$

When the point k bus arrives at the point j stop, the number of waiting trips for the longest waiting passenger on that stop is $h_{k,\ j}$, who satisfies

$$W_{k,j}(h_{k,j})>0, W_{k,j}(h_{i,j}+1)=0 \quad (2)$$

When the k car arrives at station j, the expectation of the number of people who have already waited for h cars on that station but have still not been able to board the train is $W_{k,j}(h)$; in particular, the expectation of the number of people arriving at station j to wait for a train before the kth car arrives at station j, after the (k-1) car has left the j station, is

$$W_{k,j}(0)=E\left[\int_{T_{k-1,j}}^{T_{k,j}} a_j(t)dt\right]=\int_{T_{k-1,j}}^{T_{k,j}} E\left[a_j(t)\right]dt=\int_{T_{k-1,j}}^{T_{k,j}} \mu_j(t)\,dt \quad (3)$$

When the kth vehicle leaves the jth station, the number of passengers on that vehicle is $X_{k,j}=X_{k,j}(T_{k,j})$,j=1,2,…n, which is a function of $T_{k,j}$.

When the k vehicle arrives at the j stop and the passengers have finished getting off, the number of passengers still on the vehicle is:

$$Y_{k,j}=max\left\{X_{k,j-1}-E\left[\int_{T_{k-1,j}}^{T_{k,j}} b_j(t)dt\right],0\right\}$$

$$=max\left\{X_{k,j-1}-\int_{T_{k-1,j}}^{T_{k,j}} E\left[b_j(t)dt,0\right]\right\}$$

$$=max\left\{X_{k,j-1}-\int_{T_{k-1,j}}^{T_{k,j}} v_j(t)dt,0\right\} \quad (4)$$

When the k vehicle arrives at the j stop, the maximum number of passengers that the vehicle can accept on board after the passengers have finished alighting is

$$U_{k,j}=\beta B-Y_{k,j} \quad (5)$$

That's why when

$$U_{k,j}\geq\sum_{a=h}^{h_{k,j}} W_{k,j}(a)\,,h\in N \quad (6)$$

The k bus can pick up all the passengers at the j stop.

But when

$$U_{k,j}<\sum_{a=h}^{h_{k,j}} W_{k,j}(a)\,,h\in N \quad (7)$$

The k bus cannot pick up all the passengers at the j stop, and the passengers remain

$$\sum_{a=h}^{h_{k,j}} W_{k,j}(a)-U_{k,j}, h\in N \quad (8)$$

These people will then become disgruntled.

2.4. Objective Function

The bus route optimization problem involves the interests of both passengers and bus companies, so when constructing the bus route optimization model, it is necessary to take into account both aspects, and construct an objective function that can balance the interests of both sides.

The importance of bus waiting and riding time for passengers is self-evident, they directly affect the passenger's travel experience and efficiency, so it also determines to a certain extent whether passengers choose to take the bus. Its importance is mainly reflected in the following aspects. Bus waiting and riding time directly affects the efficiency of passenger travel. If the waiting time is too long or the ride time is too long, passengers will spend more time on the road, which affects their arrangements for work, study, socialization and leisure activities. Short wait and ride times can help commuters save time and costs. Long waits can increase commuting costs, for example, if passengers wait too long for a bus, they may have to choose other modes of transportation, such as taking a taxi or driving themselves, which can increase travel costs. Shorter wait times mean that passengers can get to their destinations more quickly, reducing the inconvenience and discomfort of waiting. Riders are also more likely to choose transit systems because they provide a convenient transportation option. Shorter wait and ride times help reduce traffic congestion and vehicle emissions, thereby reducing environmental impacts. Efficient

operation of public transit systems promotes sustainable urban development and reduces problems such as air pollution and traffic noise.
Waiting time for all:

$$z_1 = \sum_{i=1}^{n} \sum_{j>i}^{n} \sum_{k=1}^{m} x_{ijk} \cdot p_{ijk} \cdot \frac{1}{2f_k} \qquad (9)$$

Everyone in car time:

$$z_2 = \sum_{i=1}^{n} \sum_{j>1}^{n} \sum_{k=1}^{m} x_{ijk} \cdot p_{ijk} (\frac{l_m}{v} + n_r \cdot t_n) \qquad (10)$$

Total passenger ride time:

$$minz = z_1 + z_2 \qquad (11)$$

At the same time, the interests of the bus companies are equally important. This is mainly reflected in the following aspects. Bus companies obtain operating funds through ticket sales and government subsidies. These funds are used to cover operating costs, including vehicle maintenance, fuel costs, employee salaries and insurance costs. If a bus company is unable to make a profit or maintain a stable financial position, the continued provision of bus services may be jeopardized. A bus company's profitability drive can motivate it to provide high-quality bus services. In order to attract more passengers, enhance competitiveness and generate more revenue, a bus company may strive to improve service quality, optimize operational efficiency and expand service coverage. The profitability of a bus company is closely related to its sustainability. Stable profitability can help a bus company to invest in upgrading its vehicles, improving its facilities, introducing advanced technology and upgrading staff training, thereby improving the quality and competitiveness of its bus services.
Company Benefits:

$$c_1 = c_s \sum_{k=1}^{m} \sum_{i=1}^{n} \sum_{j>i}^{n} x_{ijk} * l_m * f_k + c_g \left(\sum_{i=1}^{n} \sum_{j>i}^{n} \sum_{k=1}^{N} x_{i,j}^{k} * \frac{d_{ij}}{v} + 2t_p \sum_{p_{ij} \in P} z_{p_{ij}}^{k} \right) \qquad (12)$$

Ticket Revenue:

$$c_2 = \sum_{i=1}^{n} \sum_{j>i}^{n} \sum_{k=1}^{m} x_{ijk} \cdot p_{ijk} \cdot q \qquad (13)$$

Gross Revenue:

$$maxC = c_2 - c_1 \qquad (14)$$

2.5. Restrictive Condition

Each bus stop must be served by a bus, i.e., each bus stop must be assigned at least one path that is

$$d_{ij} \equiv 1 \qquad (15)$$

Bus stop level constraints:
The urban bus network consists of many bus lines, and the bus lines consist of many bus stops, so the reasonable setting of bus stop location is one of the important factors to improve the efficiency of the whole bus operation system. And bus stops and bus stops are also interrelated and interact with each other, reasonable station spacing is also one of the important factors to improve the efficiency of the bus system, and different levels of roads should be set up for different station spacing, as shown in the table below:

Table 2. Bus stop spacing for different road classes.

road class	**Urban expressways and arterial roads**	**Primary and secondary roads**	**Secondary and feeder roads**
Bus stop spacing	＞800meter	500-800meter	300-500meter

The constraint expression for the bus stop spacing $W_{i,i+1}^{r}$ is:

$$W_{min}^{r} \le W_{i,\ i+1}^{r} \le W_{max}^{r} \qquad (16)$$

Where W_{min}^{r} denotes the shortest distance between neighboring stations in line r; W_{max}^{r} denotes the longest distance between neighboring stations in line r.

Bus line length constraints:

Bus line length is set too short, will lead to residents in the bus travel can not be directly to the destination, only through the transfer or other means of transportation to reach; bus line is set too long, will make the passenger's ride time is too long, increase the driver's fatigue. So the length of bus routes should be set length limit, bus route length constraint expression:

$$l_{min} \le l_r \le l_{max} \qquad (17)$$

$$l_{max} = \frac{v \cdot T_{max}}{60} \qquad (18)$$

Where T_{max} is the maximum travel time consumption of 95 percent of urban residents, l_{min} indicates the lower limit of the length of bus routes, and l_{max} indicates the upper limit of the length of bus routes.

The state has given specific values for T_{max} for large-scale cities, medium-scale cities and small-scale cities, as well as the upper limit of the length of bus lines for each scale of cities, as shown in the table below:

Table 3. Specification of bus route lengths for different city sizes.

city scale	**number of people**	T_{MAX}**(min)**	l_{max}**(km)**
	＞Two million people	60	15
large	One to two million people	50	11.5
	＜One million people	40	10
middle		35	8.75
small		25	6.25

Bus line straightness constraint:

The ratio of the actual operating length between the first and last two stops of a bus line to the spatial straight-line length of the first and last stops is defined as the bus line non-straight-line coefficient. The size of the indicator can indicate the degree of bus bypass, taking too large a value indicates that the bus line there is a serious phenomenon of bypass, will reduce the operational efficiency of the bus, usually the value of the indicator should be smaller the better, but should be in the specified range, then the constraints on non-straight-line coefficient of the expression is:

$$\gamma_{min} \le \gamma_r \le \gamma_{max} \qquad (19)$$

$$\gamma_r = \frac{l_r}{d_r} \qquad (20)$$

Where l_r represents the actual operating length of bus line r, d_r represents the linear length of the space between the first and last stops of bus line r; γ_{max} and γ_{min} are the upper and lower limits of the non-linear coefficient of bus lines.

Different regions of the city for the bus line's non-linear coefficient requirements are different, the bus line's non-linear coefficient usually takes the value of 1.15-1.20, a single bus line's non-linear coefficient should not be greater than 1.4.

Constraints on the number of stops per bus route:

The number of stops a bus route passes through should be considered when laying out bus routes, and the number of stops passed through will directly affect the efficiency of bus operation. Passing through the number of stops will lead to too much passenger travel time is too long, passing through the number of stops is too small will result in a waste of bus capacity, then the number of stops through the line constraints expression is:

$$S_{min} \leq S \leq S_{max} \quad (21)$$

Where S_{max} and S_{min} are the maximum and minimum number of stops that a single bus route passes through, respectively.

Line network density constraint:

Bus line network density refers to the total length of all bus lines in the region and the ratio of the urban land area served by public transportation, the larger its value, indicating that residents travel more convenient, but the bus line network density is too large will lead to a higher degree of repetition of bus lines, wasting the bus capacity, so it is necessary to limit the density of the bus line network, then the constraint expression for the line network density ρ is:

$$\rho_{min} \leq \rho \leq \rho_{max} \quad (22)$$

$$\rho = \frac{\sum_{r=1}^{m} l_r}{K} \quad (23)$$

Where ρ_{max} and ρ_{min} are the maximum and minimum values of the line network density, respectively, and K is the area of the urban area served by buses.

In this paper, the multiplication and division method is used to solve the dual-objective planning problem. The multiplication and division method can simplify the multi-objective planning problem and convert multiple objective functions into one objective function, which is convenient to solve.

The final objective function using multiplication and division method is:

$$miny =$$

$$\frac{min\left[\sum_{i=1}^{n}\sum_{j>i}^{n}\sum_{k=1}^{m} x_{ijk}\cdot p_{ijk}\cdot\frac{1}{2f_k}+\sum_{i=1}^{n}\sum_{j>1}^{n}\sum_{k=1}^{m} x_{ijk}\cdot p_{ijk}(\frac{l_m}{v}+n_r\cdot t_n)+\gamma\left(\sum_{a=h}^{h_{k,j}} W_{k,j}(a)-U_{k,j}\right)+(1-\gamma)(U_{k,j}-\sum_{a=h}^{h_{k,j}} W_{k,j}(a)\right]}{max\left[\sum_{i=1}^{n}\sum_{j>i}^{n}\sum_{k=1}^{m} x_{ijk}\cdot p_{ijk}\cdot q-c_s\sum_{k=1}^{m}\sum_{i=1}^{n}\sum_{j>i}^{n} x_{ijk}*l_m*f_k+c_g\left(\sum_{i=1}^{n}\sum_{j>i}^{n}\sum_{k=1}^{N} x_{i,j}^{k}*\frac{d_{ij}}{v}+2t_p\sum_{p_{ij}\in P} z_{p_{ij}}^{k}\right)\right]}$$

$$miny = \frac{min(z_1+z_2)+\gamma\left(\sum_{a=h}^{h_{k,j}} W_{k,j}(a)-U_{k,j}\right)+(1-\gamma)(U_{k,j}-\sum_{a=h}^{h_{k,j}} W_{k,j}(a)}{max(c_2-c_1)}$$

$$min\, y = \frac{min\, z}{max\, c} \quad (24)$$

The constraints are:

$$d_{ij} \equiv 1 \quad (15)$$

$$W_{min}^{r} \leq W_{i,\ i+1}^{r} \leq W_{max}^{r} \quad (16)$$

$$l_{min} \leq l_r \leq l_{max} \quad (17)$$

$$\gamma_{min} \leq \gamma_r \leq \gamma_{max} \quad (19)$$

$$S_{min} \leq S \leq S_{max} \quad (21)$$

$$\rho_{min} \leq \rho \leq \rho_{max} \quad (23)$$

3. Case Solution Analysis

3.1. Algorithm Introduction

Dijkstra's algorithm is a greedy algorithm for solving the single-source shortest path problem. Named after Dutch computer scientist Edsger Dijkstra, the algorithm is used to find the shortest path from a single source to all other nodes, and has the advantages of guaranteed global optimality and fast computation speed over other algorithms.

3.2. Case Introduction

In this paper, the arithmetic example used to solve is the Fangchenggang City, Guangxi Province, Fangchenggang City, Fangchenggang City, as a five-tier city, the public transport business situation is not good, the number of residents who choose to take the bus for their daily trips is getting smaller and smaller, on the one hand, the increase in income of the residents, the improvement of the standard of living, there are more choices of travel modes. On the other hand, it also shows the problems of the public transportation system, can not meet the travel needs of urban residents, the need for more other modes of travel. This, the unreasonable planning of bus routes is the main reason, and not according to the development of the city, the old and old city of the turnover and timely revision of the line, which makes the original convenient to travel the bus does not play a practical role, so the city's bus routes urgently need to be optimized.

First of all, Fangchenggang Fangcheng District, Fangcheng City, each site number, and in the latitude and longitude coordinate system will be drawn out of each point, each site labeled as shown in the following table.

Table 4. Site name.

serial number	Site name	serial number	Site name
1	Xinjie	53	Municipal Court
2	Chong Lun Qiaotou	54	Military support intersection
3	Chonglun Village	55	Municipal Hospital of Traditional Chinese Medicine
4	Fangcheng Expressway Toll Station	56	Guozheng Hotel
5	Fangchenggang Management Office	57	Fangchenggang Second Bridge West
6	Hebaojiao	58	Jiale City
7	Fangchenggang Sixth Middle School	59	Renmin Road Agricultural Machinery Company
8	Middle Mountain	60	Lingfeng Temple (Second Bridge)
9	Ronggui Building Materials Wholesale Market	61	Nancheng Hotel
10	Asia-Pacific New Town	62	Fangcheng Bus Station
11	Long Island 800 Mile Uptown	63	Post and Telecommunication Office
12	Sanguankeng	64	Fangcheng District Court

13	Fangchenggang North Station	65	First Highway Passenger Transportation Center
14	Chengdong Police Station	66	Beibuwan Commercial Center
15	Huijin Asia-Pacific International	67	Printing Factory
16	Fangcheng Railway Station Square	68	Hexi Passenger Transportation Station
17	Fangcheng District Railway Station	69	Riverside Pearl
18	Sweeping Primary School	70	Cuidiyuan Intersection
19	International Plaza	71	Fangcheng District Judicial Bureau
20	Qunxing Avenue Liuyuan Intersection	72	Xinhua Bookstore
21	Taxation Bureau	73	Fangcheng District Tobacco Bureau
22	Minzu Zhongxing Intersection	74	Sunny Landscape Garden
23	City Border Guard	75	Municipal Armed Police Detachment
24	Zhongxing Intersection	76	Fangbei Intersection
25	City No.5 Middle School	77	Fangcheng District Bus Company
26	Southwest Market	78	Armed Police Internal Security Detachment
27	Josun Riverside Residence	79	Fangcheng District Education Bureau
28	Chashan Intersection	80	Fangcheng District Government
29	Shanhai Intersection (Fangqin Road)	81	Fangcheng District First Market

30	Fangcheng District Taxation Bureau	82	Shuiying intersection
31	Hengfu Plaza	83	Shuiying Village Intersection
32	Daxinan Market	84	North Gate of Fangcheng No.1 Market
33	Southwest Building Material Market	85	Fangcheng No.1 Market (Huangnigou Intersection)
34	Bay Shengjingyuan	86	Fangcheng District Third Transportation Station
35	The First People's Hospital	87	Hexi Middle School
36	Fangcheng Land Bureau	88	Fangcheng Third Bus Terminal
37	City First Hospital	89	Fangcheng No.1 Market
38	Hengfu Commercial Plaza	90	Dazhuangdang
39	Fangcheng District Land Bureau	91	Guirentang Company
40	18 Fangchenggang North Station Intersection	92	Happy Home
41	Fanggang Dongxing Intersection	93	Forestry Bureau of Fangcheng District
42	Fangcheng District Fire Brigade	94	Nongken Refined Sugar Company
43	Hengfu International Center	95	Fangcheng District Seventh Primary School

44	Fangcheng District Industry and Commerce Bureau	96	Shuiying Avenue Intersection
45	Yuan Tong Real Estate	97	Bailong Intersection (Fangdong Road)
46	Spring River Garden	98	Fangcheng District Disease Control Center (low-cost housing)
47	Chengnan Primary School	99	Hexi Gymnasium
48	Wenyuan District	100	Fangcheng District Traffic Police Brigade
49	Municipal Human Resources and Social Security Bureau	101	Huangzhutang
50	Fangcheng Second Transportation Station	102	Carp River Primary School
51	High-speed Railway Station Intersection (Golden Flower Tea Avenue)	103	Carp River Village
52	Yuantong Uptown		

The example used in this paper is Fangchenggang Fangcheng District bus station, Fangcheng District bus stops a total of 136, remove some of the more remote sites, not in this paper's optimization goals, and also does not comply with the furthest distance from the city bus stops limitations, will affect the operation of this paper's algorithm, the planning does not want the results. This paper also removes one of the stations that are very close to each other. Removing the closer neighboring stops reduces the bus stopping and waiting time for passengers, thus increasing the efficiency of bus operation. This reduces bus dwell time, increases the average speed of vehicles, reduces congestion and travel time, and improves overall operational efficiency. Reducing the number of stops reduces the cost of operating a bus route because there are fewer stops to maintain and manage. This is a cost-saving way for bus companies to help improve operational efficiency and profitability. This leaves 103 stops in the downtown area of Defense City after simplification.

According to the latitude and longitude coordinates of each bus stop, MATLAB software is used to draw each remaining stop in the same coordinate diagram. This is shown in the figure below:

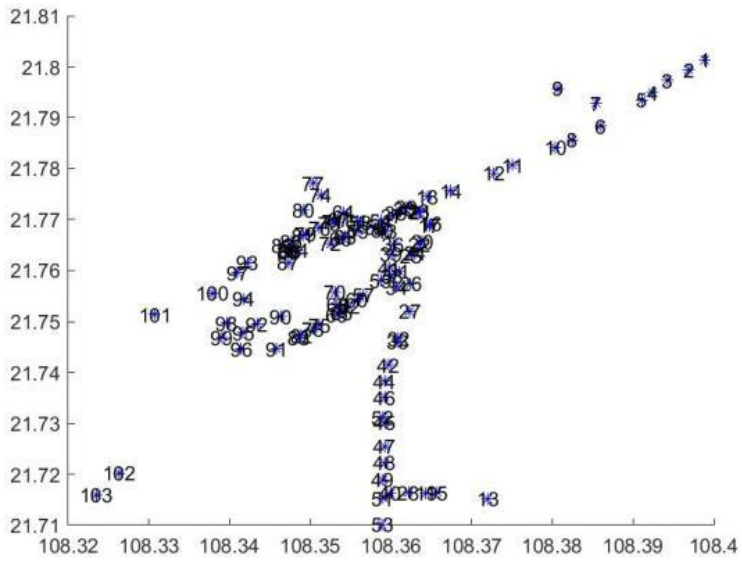

Figure 1. Map of site coordinates.

According to the above figure, among all the bus stops covered in this paper, there are bus stops far away from the urban area like 102 (Lei Yue Jiang Elementary School) and 103 (Lei Yue Jiang Village), and the bus routes passing through these two bus stops represent urban-rural buses, which makes the routes planned in this paper more realistic, and the urban-rural bus service can enhance the accessibility of transportation, especially for the residents who lack private means of transportation, such as elderly, students and low-income groups. residents, such as the elderly, students and low-income groups. It provides them with an affordable and reliable travel option that eases the difficulty of getting around.

3.3. Algorithmic Solution

Then Python software was used to design the improved Dijkstra's algorithm to optimize the bus routes in Fangchenggang City, Fangcheng District. The flowchart of the solving process is shown in the figure below:

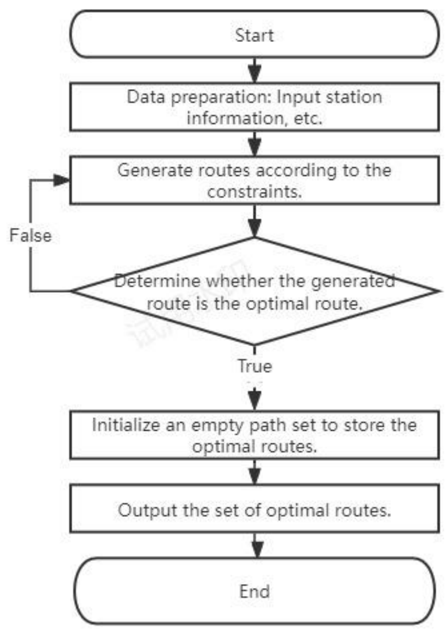

Figure 2. Flowchart of the solving process

The generated bus route planning map is shown below:

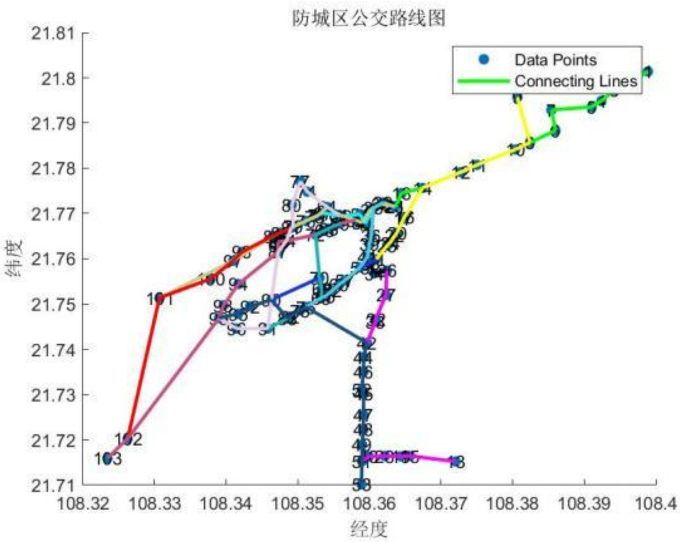

Figure 3. Bus Route Planner.

As can be seen from the above figure, the 13 bus routes pass through all 103 bus stops without missing, and reassess the layout of the existing routes, according to passenger demand and traffic flow rational planning routes, to ensure that the coverage of the main areas of the city and important nodes, while avoiding overlap and unnecessary detours to improve the efficiency of the line and the quality of service.
The stops that each line passes through are indicated by the station serial number as shown in the table below:

Table 5. Bus stops for each route.

Line code	Passing Stations
1	1-2-3-4-5-7-6-8-10-11-12-14-18-21-23
2	9-8-10-11-12-14-16-17-20-22-24-25-31-38-55-57-60
3	38-34-26-27-32-3342-44-46-52-45-47-48-49-51-53
4	13-15-19-28-40-51-49-48-47-45-52-46-44-42-33-32-27-26
5	21-29-35-43-54-56-58-67-73-79-85-93-100
6	23-30-37-50-56-61-64-71-76-79-81-89-97-101
7	74-77-80-79-81-85-89-93-97-100-101-102-103
8	56-59-63-66-72-84-86-88-87-94-98-99-102-103
9	31-38-55-57-60-62-65-69-68-70-90-92-95-99
10	59-63-66-72-70-68-69-75-78-82-83-91-96-99
11	29-30-35-36-39-41-55-57-60-62-65-69-75
12	61-64-67-71-73-76-79-81-84-87-90-92-95-99
13	53-51-49-48-47-45-52-46-44-42-75-78-82-83-90-92

3.4. Case Study

The routes mapped out in this paper, 13 in total, first satisfy the constraint of passing through every stop so that every bus stop has at least one route passing through it. The routes that pass through the most stops are the fourth and thirteenth routes, which pass through a total of 18 stops; the routes that pass through the least stops are the fifth and eleventh routes, which pass through a total of 13 stops.

The bus routes planned in this paper satisfy all the constraints.

First, it passes through all 103 bus stops, and some stops in the city are transit stops, which satisfies the actual situation in Fangchenggang City, Fangcheng District, Fangcheng City. Satisfying the

$$d_{ij} \equiv 1 \quad (15)$$

According to the criteria this paper takes $W^r_{min} = 300, W^r_{max} = 800$.Neither of the two adjacent bus stops in the same bus route is more than 800 meters, and the distance between adjacent stops on secondary roads and other roads is within 500 meters, so it is satisfied:

\

$$W^r_{min} \leq W^r_{i,\ i+1} \leq W^r_{max} \quad (16)$$

Fangchenggang City is a small city, according to the standard of a bus line's maximum length l_{max} take the value of 8.75km, the minimum length l_{min} take the value of 0. Accordingly, the use of software can also be derived from the length of each line, as shown in the table below:

Table 6. Length of each route.

Line code	Line length (km)	Line code	Line length (km)
1	5.62	8	7.34
2	5.90	9	3.65
3	5.77	10	4.02
4	6.26	11	2.98
5	4.06	12	3.80
6	4.90	13	6.23
7	8.24		

It can be seen that the longest route length planned in this paper is Line 6 with a length of 8.25km, and the shortest route is Line 5 with a length of 2.60km. satisfy the

$$l_{min} \leq l_r \leq l_{max} \quad (17)$$

Calculate the straightness of each line. First the straightness distance between the first and last two stations of each line needs to be calculated as shown in the table below:

Table 8. Distance from the first and last station of each line.

Line code	straightness
1	1.09
2	1.10
3	1.06
4	1.30
5	1.20
6	1.14
7	1.14
8	1.05
9	1.29
10	1.34
11	1.12
12	1.21
13	1.32

The thirteen routes planned in this paper have a total straight line rate of 1.18, which meets the required straight line rate of between 1.15 and 1.20 for all bus routes. And all of the individual routes have straight line rates within 1.40, which satisfies the

$$\gamma_{min} \leq \gamma_r \leq \gamma_{max} \qquad (19)$$

Fangchenggang City is a fifth-tier city, in which the number of stops on bus routes is usually limited to a certain extent, generally controlled at about 10 to 20. This range is constrained by a variety of factors including, but not limited to, city size, population density, traffic demand, road conditions, capital investment, and municipal planning. Fifth-tier cities with smaller cities tend to have fewer stations because their relatively low traffic demand does not require too many transit stations. Additionally, lower population densities may result in longer distances between stops, thus requiring longer route distances, but there is still a need to limit the number of stops in order to keep bus routes efficient and flowing. At the same time, factors such as road conditions and municipal planning can also have an impact on the design of bus routes and the setting of stops. Combined with the actual situation of Fangchenggang City, the number of bus stops passed by a single bus line is limited to 11-19, and the routes planned in this paper also meet the

$$S_{min} \leq S \leq S_{max} \qquad (21)$$

The service area of the bus routes planned in this paper is 34.52 square kilometers and the total length of the bus routes planned in this paper is 58.90 kilometers. So the density of the line network of the planned routes in this paper is 1.71km/km2.

In summary, the bus routes planned in this paper satisfy all the constraints, and although the results of our research have not yet been directly applied to real-world situations, our bus route planning scheme, which has been rigorously designed theoretically and verified through simulation, demonstrates in the virtual environment its potential to provide efficient and convenient public transportation services while satisfying all the constraints.

The comparison of the optimized routes with the previous ones is shown in the table below:

Table 9. Comparison of indicators before and after optimization.

norm	pre-optimization	post-optimization	Rate of change of indicators%
Number of public transportation stops	116	103	-11.2
Number of lines	15	13	-13.3
Average length of lines	5.56	4.53	-18.5
wire density	1.55	1.71	10.3
Non-linear coefficient	1.26	1.18	-6.3
Bus route coverage	40.42	51.18	28.2

In the evaluation index system of bus stops and routes, the length of bus routes, road network density, number of routes and non-linear coefficients in the urban area of Fangchenggang City are related to the low basic capital investment in urban public transportation, the low level of public transportation development, and irrational planning of bus routes. For the Fangchenggang City urban bus road network system in the site spacing is small, the central city site density is large, combined with the path analysis of passenger travel and multi-objective planning model, the evaluation indexes and performance changes of Fangchenggang City urban bus road network system before and after the optimization, see the table above. When the frequency of departure, passenger volume and the number of urban population remain unchanged, the number of bus stops in the central urban and suburban areas of Fangchenggang City is reduced by 13 after optimization, and the level of service and the number of passengers at the location of the bus stops and the routes traveled by the vehicles are improved, in which the coverage of the service area of the 15 bus routes is increased from 40.42% before the optimization to 51.18% after the optimization.

4. Conclusions

In this paper, for the problem of passenger dissatisfaction and bus company unprofitability caused by irrational planning of bus routes in small cities, we fully consider the factors of passenger travel time and the interests of the bus company, establish the dual-objective function of passenger travel time and the interests of the bus company, and optimize the bus routes of Fangcheng District urban area by using Dijkstra's algorithm. And with the constraints of bus stop spacing, line length, non-linear coefficient, repetition coefficient, bus network density and the number of stops each line passes through, we optimize the bus routes in order to minimize the passenger travel time and maximize the interests of the bus company.

The multi-objective customized bus route optimization model is established with the objective of minimizing the travel time of passengers and maximizing the benefits of bus companies, taking into account the interests of both passengers and bus companies. The results show that the model can effectively solve the bus route optimization problem in small cities, reduce the operating cost, and thus provide auxiliary decision-making for the public transportation management. The bus transportation efficiency is improved through rational design of bus routes.

The conclusions drawn in this article can be of certain help to the government in planning bus routes. Therefore, based on these conclusions, I put forward the following policy suggestions:

1. Adjust bus routes according to passenger satisfaction, which includes factors such as waiting time, riding time, and the degree of congestion.
2. Consider factors such as the straightness ratio of bus routes, station density, and route coverage during the initial design stage.
3. Strengthen the development of the public transportation sector and the introduction of pure electric buses to reduce carbon emissions and other types of pollution.

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Short Biography of Authors

LI Xiaolin, (Chifeng, Neimenggu, China. 2000.01.29). Bachelor's degree in Industrial Engineering from Jiangnan University, currently studying for a Master's degree in Management Science and Engineering. My research area is path optimization and urban mobility.

He currently has three internships, first as a sales intern at China Mobile, then as a personnel intern at Pilot Intelligent Equipment Manufacturing Co.

WANG Jianhua, (Wuxi, Jiangsu, China.) Highest Education/Degree: Master's Degree, Master's Degree, Title/Position: Associate Professor, Master Director. Main research interests are logistics and supply chain management and now as a purchasing intern at Wuxi-based Dr. Powertrain Co.